

# Form Factors for Exclusive Semileptonic $B$ -Decays

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## Abstract

We investigate the form factors for exclusive semileptonic decays of  $B$ -meson to  $D$ ,  $D^*$ , based on the parton picture and helped by the results of the HQET. We obtain the numerical results for the slope of the Isgur-Wise function, which is consistent with the experimental results, and we extract the dependences of hadronic form factors on  $q^2$  by varying input heavy quark fragmentation function without the nearest pole dominance ansätze.

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## 1 Introduction

$B$ -meson decay processes have been studied in detail as providing many interesting informations on the interplay of electroweak and strong interactions and as a source extracting the parameters of weak interactions, such as  $|V_{cb}|$  and  $|V_{ub}|$ . As more data will be accumulated from the asymmetric  $B$ -factories in near future, the theoretical and experimental studies on  $B$ -meson decays would also give better understandings on the Standard Model and its possible extensions.

In exclusive weak decay processes of hadrons, the effects of strong interaction are encoded in hadronic form factors. These decay form factors are Lorentz invariant functions which depend on the momentum transfer  $q^2$ , and their behaviors with varying  $q^2$  are dominated by non-perturbative effects of QCD.

Over the past few years, a great progress has been achieved in our understanding of the exclusive semileptonic decays of heavy flavors to heavy flavors [1]. In the limit where the mass of the heavy quark is taken to infinity, its strong interactions become independent of its mass and spin, and depend only on its velocity. This provides a new  $SU(2N_f)$  spin-flavor symmetry, which is not manifest in the theory of QCD. However, this new symmetry has been made explicit in a framework of the heavy quark effective theory (HQET) [2]. In practice, the HQET and this new symmetry relate all the hadronic matrix elements of  $B \rightarrow D$  and  $B \rightarrow D^*$  semileptonic decays, and all the form factors can be reduced to a single universal function, the so-called Isgur-Wise function [2, 3], which represents the common non-perturbative dynamics of weak decays of heavy mesons. However, the HQET cannot predict the values of the Isgur-Wise function over the whole  $q^2$  range, though the normalization of the Isgur-Wise function is precisely known in the zero recoil limit.

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Hence the extrapolation of  $q^2$  dependences of the Isgur-Wise function and of all form factors is still model dependent and the source of uncertainties in any theoretical studies. Therefore, it is strongly recommended to determine hadronic form factors of  $B$ -meson decay more reliably, when we think of their importance in theoretical and experimental analyses.

In this paper we develop a new approach for exclusive semileptonic  $B$  decays to  $D$ ,  $D^*$ , and predict the  $q^2$  dependences of all form factors, inspired by the parton model. Previously the parton model approach has been established to describe inclusive semileptonic  $B$  decays [4, 5], and found to give excellent agreements with experiments for electron energy spectrum at all energies. The parton model approach for the inclusive decays pictures the mesonic decay as the decay of the partons in analogy to deep inelastic scattering process. The probability of finding a  $b$ -quark in a  $B$  meson carrying a fraction  $x$  of the meson momentum in the infinite momentum frame is given by the distribution function  $f(x)$ .

While many attempts describing exclusive  $B$  decays often take the pole-dominance ansätze as behaviors of form factors with varying  $q^2$ , in our approach they are derived by the kinematical relations between initial  $b$  quark and final  $c$  quark. According to the Wirbel *et al.* model [6], which is one of the most popular model to describe exclusive decays of  $B$  mesons, the hadronic form factors are related to the meson wavefunctions' overlap-integral in the infinite momentum frame, but in our model they are determined by integral of the distribution functions of quarks inside the mesons.

In Section II, we develop the parton model approach for exclusive semileptonic decays of  $B$  meson. All the theoretical details for  $B \rightarrow D l \nu$  and  $B \rightarrow D^* l \nu$  are given in Section III. Section IV contains discussions and conclusions of this paper.

## 2 Parton Model Approach for Exclusive Decays of B Meson

We now develop the parton model approach for exclusive semileptonic decays of  $B$  meson by extending the previously established inclusive parton model, and by combining with the results of the HQET. Theoretical formulation of this approach is, in a sense, related to Drell-Yan process, while the parton model of inclusive  $B$  decays is motivated by deep inelastic scattering process. And the bound state effects of exclusive  $B$  decays are encoded into the hadronic distribution functions of partons inside an initial  $B$  meson and of partons of a final state resonance hadron. In Fig. 1, we show the schematic diagrams of Drell-Yan process and the related exclusive semileptonic decay of  $B$  meson. We assume that the Lorentz invariant hadronic decay width can be obtained using the structure functions,

$$E_B \cdot d\Gamma(B \rightarrow D(D^*) e \nu) = \int dx \int dy f_B(x) E_b \cdot d\Gamma(b \rightarrow c e \nu) f_D(y) \quad (1)$$

following the Drell-Yan case. The first integral represents the effects of motion of  $b$  quark within  $B$  meson and the second integral those of  $c$  quark within  $D$  meson. The variables  $x$  and  $y$  are defined as fractions of momenta of partons to momenta of mesons,

$$p_b = x p_B, \quad p_c = y p_D, \quad (2)$$

in the infinite momentum frame,  $|\mathbf{p}| \gg m$ . In this frame, the eq. (2) is valid for the four momenta of massive partons. The functions  $f_B(x)$  and  $f_D(y)$  are the distribution function of  $b$  quark inside  $B$  meson, and the fragmentation function of  $c$  quark to  $D$  meson, respectively, which are also defined in that frame. Since the momentum fractions and the distribution functions are all defined in the infinite momentum frame, we have to calculate the Lorentz invariant quantity,  $E \cdot d\Gamma$  as defined in Eq. (1), to use at any other frame safely.

The distribution function can be identified with the fragmentation function for a fast moving  $b$ -quark to hadronize into a  $B$  meson in the infinite momentum frame. In general the distribution and fragmentation functions of a heavy quark ( $Q = t, b, c$ ) in a heavy meson ( $Qq$ ), which are closely related by a time reversal transformation, are of very similar functional forms, and peak both at large value of  $x$ . Brodsky *et al.* [7] have calculated the distribution function of a heavy quark, which has the same form as Peterson's fragmentation function [8]. Therefore, here we follow the previous work of Paschos *et al.* [4] to use the Peterson's fragmentation function for both distributions,  $f_B(x)$  and  $f_D(y)$ . It has the functional form:

$$f_Q(z) = N_Q z^{-1} \left( 1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z} \right)^{-2}, \quad (3)$$

where  $N_Q$  is a normalization constant, and  $Q$  denotes  $b$  or  $c$  quark. This functional form is not purely ad hoc., but motivated by general theoretical arguments, that the transition amplitude for a fast moving heavy quark  $Q$  to fragment into a heavy meson ( $Qq$ ) is proportional to the inverse of the energy transfer  $\Delta E^{-1}$ .

This function (3) has a peak at  $z \sim 1 - \sqrt{\epsilon_Q}$ . Since the parameter  $\epsilon_Q$  is related to the ratio of the effective light quark mass to the heavy quark mass  $\sim m_q^2/m_Q^2$ , the peak approaches to 1 in the limit of  $m_Q \rightarrow \infty$ , and the function shows the similar behavior to  $\delta(1-z)$ . Hence we find that this functional form is supported by the HQET in this indirect manner.

In the Drell-Yan process, the rest degrees of freedom of initial nucleons which do not take part in the scattering make incoherent final states, see in Fig. 1 (a). In the exclusive semileptonic decay of a heavy meson into a final state heavy meson, however, two sets of left-over light-degrees of freedom are summed to have the connection, i.e. see Fig. 2 (b). To connect them we need a relation between  $x$  and  $y$  from the decay kinematics. The momentum transfer of the decay between mesons is defined as

$$q \equiv p_B - p_D. \quad (4)$$

On the other hand, the momentum transfer of the partonic subprocess is given by

$$\begin{aligned} q^{(\text{parton})} &= p_b - p_c \\ &= xp_B - yp_D. \end{aligned} \quad (5)$$

Note that the second line of the Eq. (5) holds only in the infinite momentum frame while Eq. (4) holds at any frames. In fact, the heavy meson's momentum would be  $p_H = p_Q + k + \mathcal{O}(1/m_Q)$ , where  $H = B, D$  or  $D^*$ , and  $Q = b$  or  $c$ . And  $k$  denotes the momentum of the light-degrees of freedom, and has the size of the effective mass of a common light degree of freedom,  $\bar{\Lambda}$ . Therefore we have  $q = q^{(\text{parton})}$  up to the common part of the  $1/m_Q$  corrections in the infinite momentum frame, and with these kinematic relations, (4) and (5), we derive the following relation

$$y(x, q^2) = \frac{1}{m_D} \sqrt{x(x-1)m_B^2 + (1-x)q^2 + xm_D^2}. \quad (6)$$

Note that if  $x = 1$ ,  $y = 1$ , this equation describes the decay of the free quark with mass  $m_B$  to the final quark with mass  $m_D$ . Substituting  $y$  of Eq. (1) for  $y(x, q^2)$  of Eq. (6), the double integral of Eq. (1) is reduced to the single integral over  $x$ . Using this relation, we can sum the intermediate states in Fig. 1(b).

We note here that the kinematic relation (6) are valid approximations for the heavy-to-heavy resonance decays, with the common light-degrees of freedom of the size  $\mathcal{O}(1/m_Q)$ . As explained before, in the limit where  $f_Q(x) = \delta(1-x)$  by increasing  $m_Q$  to infinity, we can reproduce the HQET leading term. By comparison, the inclusive parton model approach is more reliable for the heavy-to-light non-resonant decays to final states of many particles.

### 3 Form Factors for semileptonic $B \rightarrow D(D^*)$ Decays

#### 3.1 $B \rightarrow De\nu$

From Lorentz invariance we write the matrix element of the decay  $\bar{B} \rightarrow De\bar{\nu}$  in the form

$$\langle D|J_\mu|B \rangle = f_+(q^2)(p_B + p_D)_\mu + f_-(q^2)(p_B - p_D)_\mu , \quad (7)$$

and in terms of the HQET

$$\langle D(v')|J_\mu|B(v) \rangle = \sqrt{m_B m_D} (\xi_+(v \cdot v')(v + v')_\mu + \xi_-(v \cdot v')(v - v')_\mu) . \quad (8)$$

Due to the conservation of leptonic currents, the form factors multiplied by  $q_\mu$  do not contribute. Then the hadronic tensor is given by

$$\begin{aligned} H_{\mu\nu} &= \langle D|J_\mu|B \rangle \langle B|J_\nu^\dagger|D \rangle \\ &= 2 |f_+(q^2)|^2 (p_{B\mu} p_{D\nu} + p_{B\nu} p_{D\mu}) , \end{aligned} \quad (9)$$

and can be expressed by the Isgur-Wise function,

$$H_{\mu\nu} = R^{-1} |\xi(v \cdot v')|^2 (p_{B\mu} p_{D\nu} + p_{B\nu} p_{D\mu}) \left( 1 + \mathcal{O}\left(\frac{1}{m_Q}\right) \right) , \quad (10)$$

where

$$R = \frac{2\sqrt{m_B m_D}}{m_B + m_D} .$$

The partonic subprocess decay width for  $B \rightarrow De\nu$  decay is given by

$$\begin{aligned} E_b \cdot d\Gamma(b \rightarrow ce\nu) &= \frac{1}{2} (2\pi)^4 \delta^4(p_b - p_c - q^{(\text{parton})}) \cdot 2G_F^2 |V_{cb}|^2 H_{\mu\nu}^{(\text{parton})} L^{\mu\nu} \\ &\quad \times \frac{d^3 p_c}{(2\pi)^3 2E_c} \frac{d^3 p_e}{(2\pi)^3 2E_e} \frac{d^3 p_\nu}{(2\pi)^3 2E_\nu} , \end{aligned} \quad (11)$$

where

$$\begin{aligned} L^{\mu\nu} &= 2 (p_e^\mu p_\nu^\nu + p_e^\nu p_\nu^\mu - g^{\mu\nu} p_e \cdot p_\nu + i\epsilon^{\mu\nu\alpha\beta} p_{e\alpha} p_{\nu\beta}) , \\ H_{\mu\nu}^{(\text{parton})} &= N (p_{b\mu} p_{c\nu} + p_{b\nu} p_{c\mu}) . \end{aligned} \quad (12)$$

Here  $H_{\mu\nu}^{(\text{parton})}$  denotes the partonic subprocess' hadronic tensor contributing  $B \rightarrow D$  decay. Now we need to discuss about it. In the inclusive decays  $B \rightarrow X_c e \nu$ , the hadronic tensor is given by

$$W_{\mu\nu} = 2 (p_{b\mu} p_{c\nu} + p_{b\nu} p_{c\mu} - g_{\mu\nu} p_b \cdot p_c - i\epsilon_{\mu\nu\alpha\beta} p_b^\alpha p_c^\beta) .$$

As can be easily seen, this hadronic tensor contains all the possible spin configurations of  $b \rightarrow c$  transition from spin 0 state. Here we are interested in only  $B \rightarrow D$  process, where the spin does not change during the process. Therefore, we have to choose only the spin-inert part which contributes to  $B \rightarrow D$  process from the inclusive hadronic tensor,  $W_{\mu\nu}$ . By comparing with Eq. (10), we find that the spin-inert part has the form of  $(p_{b\mu} p_{c\nu} + p_{b\nu} p_{c\mu})$ , as shown in (12). Besides, we do not know how much spin-inert part really contributes to  $B \rightarrow D$  semileptonic decay, because other decays such as  $B \rightarrow D^*$ ,  $B \rightarrow D^{**}$  also contain spin-inert parts. Therefore, the parameter  $N$  is introduced

to estimate the size of spin-inert part which contributes to  $B \rightarrow D$  process. The constant  $N$  will be later determined by the zero recoil limit of the Isgur-Wise function.

Using the relation (2), we can write the hadronic tensor in the parton level as follows

$$H_{\mu\nu}^{(\text{parton})} = N \, xy \, (p_{B\mu} p_{D\nu} + p_{B\nu} p_{D\mu}) \quad . \quad (13)$$

The momentum conservation of the partonic subprocess corresponds to the momentum conservation in the hadronic level in our model, as explained before. So we can substitute the Dirac delta function  $\delta^4(p_b - p_c - q^{(\text{parton})})$  for  $\delta^4(p_B - p_D - q)$  in Eq. (11) with no loss of generality. Therefore, we write the decay width of  $\bar{B} \rightarrow De\bar{\nu}$ ,

$$\begin{aligned} E_B \cdot d\Gamma(B \rightarrow De\nu) &= \int dx \, f_B(x) \, f_D(y(x, q^2)) E_b \, d\Gamma(b \rightarrow ce\nu) \\ &= \int dx \, f_B(x) f_D(y(x, q^2)) (2\pi)^4 \delta^4(p_B - p_D - q) \\ &\quad \times G_F^2 |V_{cb}|^2 N \, xy(x, q^2) (p_{B\mu} p_{D\nu} + p_{B\nu} p_{D\mu}) L^{\mu\nu} \\ &\quad \times y^2(x, q^2) \frac{d^3 p_D}{(2\pi)^3 2E_D} \frac{d^3 p_e}{(2\pi)^3 2E_e} \frac{d^3 p_\nu}{(2\pi)^3 2E_\nu} \quad . \end{aligned} \quad (14)$$

Hence we find the hadronic tensor

$$\begin{aligned} H_{\mu\nu}(B \rightarrow De\nu) &= N \int dx \, f_B(x) f_D(y(x, q^2)) \, xy^3(x, q^2) (p_{B\mu} p_{D\nu} + p_{B\nu} p_{D\mu}) \\ &\equiv N \, \mathcal{F}(q^2) (p_{B\mu} p_{D\nu} + p_{B\nu} p_{D\mu}) \quad , \end{aligned} \quad (15)$$

where we factorize the function  $\mathcal{F}(q^2)$  defined as

$$\mathcal{F}(q^2) \equiv \int dx \, f_B(x) f_D(y(x, q^2)) \, xy^3(x, q^2) \quad . \quad (16)$$

For given  $q^2$  in our parton picture, the function  $\mathcal{F}(q^2)$  measures the weighted transition amplitude, which is explicitly related to the overlap integral of distribution functions of initial and final state hadrons.

Comparing (15) with the Eq. (10), the Isgur-Wise function is calculated within the parton model approach

$$|\xi(v \cdot v')|^2 \left( 1 + \mathcal{O}\left(\frac{1}{m_Q}\right) \right) = 4N \cdot R \cdot \mathcal{F}(v \cdot v') \quad . \quad (17)$$

As explained before, in our model the non-perturbative QCD effects are included through the distribution functions, so the function  $\mathcal{F}(q^2)$  contains all orders of  $1/m_Q$  corrections. Note that, however, our model ignores the effects of the transverse quark motion and off-mass shell, the full  $1/m_Q$  effects are not being considered. From the value of the zero recoil limit of the Isgur-Wise function with  $1/m_Q$  corrections [9], we can determine the numerical value of  $N$ . In Table 1, we show the numerical values of  $N$  with varying the parameters of the fragmentation functions,  $(\epsilon_b, \epsilon_c)$ . Unfortunately we cannot obtain the analytic form of  $\mathcal{F}(q^2)$  due to the nontriviality of the integrals of  $f(x)$ . However, we can numerically obtain the behaviors of the Isgur-Wise function with varying  $q^2$  from the Eq. (17), and calculate its slope parameter. The normalization constant  $N$  and the slope parameters  $\rho^2$  with the input values of  $(\epsilon_b, \epsilon_c)$  are also shown in the Table 1. We varied  $\epsilon_b$  from 0.004 to 0.006, and  $\epsilon_c$  from 0.04 to 0.1. As explained earlier, the parameter  $\epsilon_Q$  is related to the ratio of the effective light quark mass to the heavy quark mass  $\sim m_q^2/m_Q^2$ . And therefore, our

input values of  $(\epsilon_b, \epsilon_c)$  correspond to the range  $m_c^2/m_b^2 = 0.04 \sim 0.15$ , which is consistent with the prediction of the HQET [1]. For more details on the comparison with the HQET, see Section IV.

The  $q^2$  spectrum is given by

$$\frac{d\Gamma(\bar{B} \rightarrow De\bar{\nu})}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{96\pi^3 m_B^3} \mathcal{F}(q^2) N \left( (m_B^2 - m_D^2 + q^2)^2 - 4m_B^2 q^2 \right)^{3/2} . \quad (18)$$

And in Fig. 2(a), our prediction is shown with the parameters  $(\epsilon_b = 0.004, \epsilon_c = 0.04)$ , compared with Wirbel *et al.*'s model prediction [6], which shows quite a good agreement.

### 3.2 $B \rightarrow D^* e \nu$

We write the matrix element of the decay  $\bar{B} \rightarrow D^* e \bar{\nu}$  in familiar form

$$\begin{aligned} \langle D^* | V_\mu + A_\mu | B \rangle &= \frac{2i}{m_B + m_{D^*}} \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_{D^*}^\alpha p_B^\beta V(q^2) \\ &+ (m_B + m_{D^*}) \epsilon^*{}_\mu A_1(q^2) - \frac{\epsilon^* \cdot q}{m_B + m_{D^*}} (p_B + p_{D^*})_\mu A_2(q^2) \\ &- 2m_{D^*} \frac{\epsilon^* \cdot q}{q^2} q_\mu A_3(q^2) + 2m_{D^*} \frac{\epsilon^* \cdot q}{q^2} q_\mu A_0(q^2) , \end{aligned} \quad (19)$$

where

$$A_3(q^2) = \frac{m_B + m_{D^*}}{2m_{D^*}} A_1(q^2) - \frac{m_B - m_{D^*}}{2m_{D^*}} A_2(q^2) ,$$

and in terms of the HQET

$$\begin{aligned} \langle D^*(v') | V_\mu + A_\mu | B(v) \rangle &= \sqrt{m_B m_{D^*}} \\ &\times (i\xi_V (v \cdot v') \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} v'^\alpha v^\beta v(q^2) + \xi_{A_1} (v \cdot v') (v \cdot v' + 1) \epsilon^*{}_\mu \\ &- \xi_{A_2} (v \cdot v') \epsilon^* \cdot v v_\mu - \xi_{A_3} (v \cdot v') \epsilon^* \cdot v v'_\mu) . \end{aligned} \quad (20)$$

In fact,  $A_0(q^2)$  and  $A_3(q^2)$  do not contribute here because of leptonic currents conservation. With the heavy quark expansion, the hadronic form factors are related to the Isgur-Wise function  $\xi(v \cdot v')$ , such as  $\xi_i(v \cdot v') = \xi(v \cdot v')(\alpha_i + \mathcal{O}(1/m_Q))$ , where  $\alpha_{A_2} = 0$  and  $\alpha_i = 1$  otherwise.

The hadronic tensor of  $\bar{B} \rightarrow D^* e \bar{\nu}$  decay is obtained using the HQET

$$\begin{aligned} H_{\mu\nu} &= \sum_{\text{spin}} \langle D^* | V_\mu + A_\mu | B \rangle \langle D^* | V_\nu + A_\nu | B \rangle^* \\ &= R^{*-1} |\xi(v \cdot v')|^2 \left[ \left( 1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) (p_{B\mu} p_{D^*\nu} + p_{B\nu} p_{D^*\mu}) (1 + \mathcal{O}(\frac{1}{M_Q})) \right. \\ &\quad \left. - 2 \left( 1 - \frac{q^2}{(m_B + m_{D^*})^2} \right) \left( g_{\mu\nu} p_B \cdot p_{D^*} (1 + \mathcal{O}(\frac{1}{M_Q})) - i \epsilon_{\mu\nu\alpha\beta} p_B^\alpha p_{D^*}^\beta (1 + \mathcal{O}(\frac{1}{M_Q})) \right) \right] , \end{aligned} \quad (21)$$

where  $R^* = 2\sqrt{m_B m_{D^*}}/(m_B + m_{D^*})$ . Investigating the above relation, we can find the form of the hadronic tensor for the partonic subprocess which contributes to the decay  $\bar{B} \rightarrow D^* e \bar{\nu}$ . The resulting hadronic tensor is written down in the form,

$$\begin{aligned} H_{\mu\nu}^{(\text{parton})} &= \left( 1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) N_1 (p_{b\mu} p_{c\nu} + p_{b\nu} p_{c\mu}) \\ &- 2 \left( 1 - \frac{q^2}{(m_B + m_{D^*})^2} \right) (N_2 g_{\mu\nu} p_b \cdot p_c - i N_3 \epsilon_{\mu\nu\alpha\beta} p_b^\alpha p_c^\beta) . \end{aligned} \quad (22)$$

The parameters  $N_i$ 's give the relative size of form factors and overall normalization. In general they are not constants and have the  $q^2$  dependences. In the heavy quark limit,  $N_i$ 's become constants and the values are equal to that of the normalization constant  $N$  in  $B \rightarrow D e \nu$  process.

In order to investigate the procedure more conveniently, we define the ratios of form factors as follows:

$$\begin{aligned} R_1 &\equiv \left(1 - \frac{q^2}{(m_B + m_{D^*})^2}\right) \frac{V(q^2)}{A_1(q^2)} , \\ R_2 &\equiv \left(1 - \frac{q^2}{(m_B + m_{D^*})^2}\right) \frac{A_2(q^2)}{A_1(q^2)} , \end{aligned} \quad (23)$$

where  $V(q^2)$ ,  $A_1(q^2)$  and  $A_2(q^2)$  denote vector and axial vector form factors respectively. Then we can write the relations among form factors and the Isgur-Wise function as

$$\begin{aligned} A_1(q^2) &= \left(1 - \frac{q^2}{(m_B + m_{D^*})^2}\right) R^{*-1} \xi(q^2) , \\ A_2(q^2) &= R_2 R^{*-1} \xi(q^2) , \\ V(q^2) &= R_1 R^{*-1} \xi(q^2) . \end{aligned} \quad (24)$$

Note that the Isgur-Wise function  $\xi(v \cdot v')$  in these expressions contains full  $1/m_Q$  corrections and it corresponds to  $\hat{\xi}(v \cdot v')$  in the Ref. [9], which is normalized at zero recoil up to the second order corrections  $\hat{\xi}(1) = 1 + \delta_{1/m_Q}$ . And  $R_1$  and  $R_2$  become to be unity in the heavy quark limit, and Neubert's estimates of the  $1/m_Q$  corrections [1] give

$$R_1 \approx 1.3 , \quad R_2 \approx 0.8 , \quad (25)$$

which are model dependent. Recently CLEO [10] obtained the values of the parameters  $R_1$ ,  $R_2$  by fitting them with the slope parameter of the Isgur-Wise function  $\rho^2$ . Since  $R_1$  and  $R_2$  have very weak  $q^2$  dependence, in the CLEO analysis they approximately obtained the  $q^2$  independent values,

$$\begin{aligned} R_1 &= 1.30 \pm 0.36 \pm 0.16 , \\ R_2 &= 0.64 \pm 0.26 \pm 0.12 . \end{aligned} \quad (26)$$

Hereafter we also take them to be constants for simplicity.

In our model the parameters  $N_1$ ,  $N_2$  and  $N_3$  are represented in terms of  $R_1$  and  $R_2$  as follows,

$$\begin{aligned} N_1 &= \frac{N/2}{1 - 2q^2/(m_B + m_{D^*})^2} \left[ -\frac{q^2}{(m_B + m_{D^*})^2} \cdot 2R_1^2 \right. \\ &\quad \left. + \left(1 - \frac{q^2}{(m_B + m_{D^*})^2}\right) \left( \frac{(1 + R_2)^2}{2} + \frac{m_B^2 + q^2}{2m_{D^*}^2} (1 - R_2)^2 + \frac{2m_B m_{D^*} - q^2}{2m_{D^*}^2} (1 - R_2^2) \right) \right] , \\ N_2 &= \frac{N}{2} \left[ (1 + R_1^2) + \frac{2m_B m_{D^*}}{m_B^2 + m_{D^*}^2 - q^2} (1 - R_1^2) \right] , \\ N_3 &= N R_1 . \end{aligned} \quad (27)$$

In the heavy quark limit, we know that  $R_1 = R_2 = 1$ . Using the expression  $R_i = 1 + \mathcal{O}(1/m_Q)$  we can separate the leading contributions and  $1/m_Q$  corrections in  $N_i$ 's:

$$\begin{aligned} N_1 &= \frac{N}{4} (1 + R_2)^2 + \mathcal{O}(1 - R_2) , \\ N_2 &= \frac{N}{2} (1 + R_1^2) + \mathcal{O}(1 - R_1) , \\ N_3 &= N R_1 . \end{aligned} \quad (28)$$

When  $R_1, R_2 \rightarrow 1$ , we explicitly see that  $N_1 = N_2 = N_3 \rightarrow N$ .

Now substituting  $\xi(v \cdot v')$  for  $\mathcal{F}(q^2)$  with the Eq. (17), the hadronic tensor for the decay  $\bar{B} \rightarrow D^* e \bar{\nu}$  is given by

$$H_{\mu\nu}(B \rightarrow D^*) = \mathcal{F}(q^2) \left[ \left(1 - \frac{2q^2}{(m_B + m_{D^*})^2}\right) N_1 (p_{B\mu} p_{D^*\nu} + p_{B\nu} p_{D^*\mu}) \right. \\ \left. - 2 \left(1 - \frac{q^2}{(m_B + m_{D^*})^2}\right) (N_2 g_{\mu\nu} p_B \cdot p_{D^*} - i N_3 \epsilon_{\mu\nu\alpha\beta} p_B^\alpha p_{D^*}^\beta) \right] . \quad (29)$$

The  $q^2$  dependences of form factors are mainly determined by the function  $\mathcal{F}(q^2)$ , instead of the commonly used pole-dominance ansätze.

When we calculate the hadronic tensor within the HQET framework, we have generally some parameters parametrizing non-perturbative effects, which are obtained in model dependent ways. The slope parameter is such a characteristic parameter of the Isgur-Wise function, which represents the common behaviors of form factors. We calculated it, and find that the value of the slope parameter is related to the parameters  $\epsilon_b$  and  $\epsilon_c$  in our approach. The HQET also contains the parameter  $\lambda_1 \sim -\langle k_Q^2 \rangle$  which is related to the kinetic energy of the heavy quark inside the heavy meson [11], and spin-symmetry breaking term  $\lambda_2 = \frac{1}{4}(m_V^2 - m_P^2)$  corresponding to the mass splitting of pseudoscalar mesons and vector mesons. In our approach, we have two parameters  $R_1$  and  $R_2$  correspondingly, which are experimentally measurable. Also the values of  $\epsilon_b$  and  $\epsilon_c$  can be independently determined from the various methods experimentally and theoretically. We use the values for  $R_1$  and  $R_2$  from the CLEO fit results of Ref. [10], and for  $\epsilon_b$  and  $\epsilon_c$  from Ref. [8, 12].

Finally, we obtain the decay spectrum

$$\frac{d\Gamma(\bar{B} \rightarrow D^* e \bar{\nu})}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^5} \mathcal{F}(q^2) \left( (m_B^2 - m_{D^*}^2 + q^2)^2 - 4m_B^2 q^2 \right)^{1/2} \\ \times \left[ m_B^2 W_1(q^2) ((m_B^2 - m_{D^*}^2 + q^2)^2 - m_B^2 q^2) \right. \\ \left. + \frac{3}{2} m_B^2 W_2(q^2) (m_B^2 - m_{D^*}^2 + q^2) + 3m_B^2 W_3(q^2) \right] , \quad (30)$$

where

$$W_1(q^2) = -N_1 \left( 1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) , \\ W_2(q^2) = N_1 (m_B^2 - m_{D^*}^2 + q^2) \left( 1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) - 2N_3 q^2 \left( 1 - \frac{q^2}{(m_B + m_{D^*})^2} \right) , \\ W_3(q^2) = -N_1 m_B^2 q^2 \left( 1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) \\ + N_3 q^2 (m_B^2 - m_{D^*}^2 + q^2) \left( 1 - \frac{q^2}{(m_B + m_{D^*})^2} \right) \\ + N_2 q^2 (m_B^2 + m_{D^*}^2 - q^2) \left( 1 - \frac{2q^2}{(m_B + m_{D^*})^2} \right) , \quad (31)$$

and  $\mathcal{F}(q^2)$  is defined in (16). The result is plotted in Fig. 2 (b), also compared with the CLEO data [10]. The thick solid line is our model prediction with the parameters ( $\epsilon_b = 0.004$ ,  $\epsilon_c = 0.04$ ) and the dashed line the Wirbel *et al.* model prediction [6].



## 4 Discussions and Conclusions

All form factors show the common behavior for varying  $q^2$ , which is described by the Isgur-Wise function of the HQET, which represents the common non-perturbative dynamics of weak decays of heavy mesons. Ever since Fakirov and Stech [13], the nearest pole dominance has been usually adopted as the dependence of common behaviors on  $q^2$ . In our approach, their  $q^2$  dependences are extracted from the kinematic relations of  $b$ - and  $c$ -quark. When  $b$ -quark decays to  $c$ -quark, the momentum transfer to leptonic sector is equal to the difference between  $b$ -quark momentum and  $c$ -quark momentum in the parton picture. The  $b$ - and  $c$ -quark momenta within the  $B$  and  $D$  mesons have some specific distributions. For given momentum transfer  $q^2$ , there exist possible configurations of  $b$ - and  $c$ -quark momentum pairs ( $p_b$ ,  $p_c$ ), and each pair is appropriately weighted with the momentum distributions of the quarks. Our  $\mathcal{F}(q^2)$  function in (16) measures the weighted transition amplitude for given  $q^2$  in the parton picture; it is explicitly given by the overlap integral of distribution functions of initial and final state hadrons. This is common to all form factors, as explained in Section II.

As mentioned earlier, the non-perturbative strong interaction effects are also considered through the distribution functions in our model, so  $\xi(v \cdot v')$  obtained from Eq. (17) corresponds to the hadronic form factor  $\hat{\xi}(v \cdot v')$  defined in the Ref. [9], including  $1/m_b$  corrections rather than the lowest order Isgur-Wise function. And the slope parameter of our results in Table 1 also corresponds to  $\hat{\rho}^2$  related to  $\hat{\xi}(v \cdot v')$ . We obtain the values of the slope parameter  $\rho$  within the parton model framework, as in Table 1,

$$\rho^2 = 0.552 - 0.858 \quad ,$$

which are compatible with the Neubert's prediction [9],

$$\hat{\rho}^2 \simeq \rho^2 \pm 0.2 = 0.7 \pm 0.2 \quad .$$

Our result is rather smaller than the predictions of other models,

$$\rho^2 = 1.29 \pm 0.28 \quad [14] \quad ,$$

$$\rho^2 = 0.99 \pm 0.04 \quad [15] \quad ,$$

but it is consistent with the average value measured by experiments [16],

$$\hat{\rho}^2 = 0.87 \pm 0.12 \quad .$$

In calculating the numerical values, we still have two free parameters  $\epsilon_b$  and  $\epsilon_c$  of Eq. (6), *i.e.* of the heavy quark fragmentation functions. Their values can be determined independently from the various experimental and theoretical methods<sup>5</sup>. For the parton model to be consistent with the HQET, we require that with the fixed value of the parameter  $\epsilon_Q$ , all the appropriate results of the parton model approach agree with those of the HQET. In other words, the value of parameter  $\epsilon_Q$  should be determined to give all the phenomenological results to coincide with those of the HQET. In this point of view, we have previously studied the parton model approach for inclusive semileptonic decays of  $B$  meson in the Ref. [17], and showed that the value  $\epsilon_b \approx 0.004$  gives consistent results with those of the HQET. In this paper we use the value of  $\epsilon_b$  as 0.004 or 0.006. The latter value is given by the experiments for the determination of the Peterson fragmentation function [12]. We find that our prediction of the slope parameter  $\rho^2$  with the parameter  $\epsilon_b = 0.004$

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<sup>5</sup>Theoretically, it is interesting to calculate  $f_Q(z) = \delta(1-z) + \mathcal{O}(1/m_Q^2)$  within the HQET, which can reproduce the phenomenological predictions.

and  $\epsilon_c = 0.04$  gives the best agreed value  $\sim 0.7$  with that of the HQET,  $\hat{\rho}^2 = 0.7 \pm 0.2$ . In this context, we conclude that our model with the parameter  $\epsilon_b = 0.004$  gives consistent predictions with the HQET.

To investigate the possible model dependences of our approach on the input fragmentation functions, we now study the cases with other fragmentation functions. Following the Artru–Mennessier string model [18] for heavy flavors, we have the fragmentation function such as

$$f_Q(z) = N_Q \frac{(1-z)^a}{z^{1+bm_Q^2}} \exp\left(-\frac{bm_Q^2}{z}\right), \quad (32)$$

with parameters  $b = 0.8 \text{ GeV}^{-2}$  and  $a \approx 0.5$ , and the  $c$ - and  $b$ -quark masses identified with the masses of the lowest-lying vector meson states [19]. Another fragmentation function is recently developed with the help of the perturbative QCD (PQCD). Inspired by the PQCD of the heavy quarkonium, the functional form for the heavy-light mesons [20] is derived as

$$f_Q(z) = N_Q \frac{r_Q z (1-z)^2}{(1 - (1-r_Q)z)^6} \left( 6 - 18(1-2r_Q)z + (21 - 74r_Q + 68r_Q^2)z^2 - 2(1-r_Q)(6 - 19r_Q + 18r_Q^2)z^3 + 3(1-r_Q)^2(1-2r_Q + 2r_Q^2)z^4 \right), \quad (33)$$

with the free parameter  $r_Q$  being the mass relation  $m_Q/(m_Q + m_q)$ , and with the light quark mass  $m_q$ . We call this function the PQCD inspired fragmentation function. With these two fragmentation functions, we obtain the numerical values of  $\rho^2 = 1.124$  from the string fragmentation function, and  $\rho^2 = 0.605$  from the PQCD inspired fragmentation function. The results are shown in Table 2 compared with those with Peterson’s function. As can be seen, the string fragmentation function gives a somewhat larger value of  $\rho$ , but the ‘recently developed’ PQCD inspired function gives a similar value to that of Peterson fragmentation function.

Phenomenologically, our model prediction on  $q^2$  spectrum in the  $B \rightarrow D^* e \nu$  decay shows a good agreement with the result of the CLEO [10], as shown in Fig. 2(b). If we let  $f_Q(z) = \delta(1-z)$  and  $R_1 = R_2 = 1$  in our model, we can reproduce the lowest order results of the HQET, and obtain the similar plot with those of other models in Fig. 2(b). For the  $B \rightarrow D e \nu$  decay, our results agree with those of other models, as in Fig. 2(a).

Finally we obtain the ratio of integrated total widths  $\Gamma(B \rightarrow D^*)/\Gamma(B \rightarrow D) \approx 2.66$ , which agrees with the experimental results [21]. It may be a phenomenological support of our model because this quantity is independent of the CKM elements  $|V_{cb}|$ , which has uncertainties in determining its value yet. The perturbative QCD corrections can be factorized in the decay width calculation [9, 22], which does not affect the ratio  $\Gamma(B \rightarrow D^*)/\Gamma(B \rightarrow D)$ .

To summarize, we investigated the form factors for exclusive semileptonic decays of  $B$ -meson to  $D$ ,  $D^*$ , based on the parton picture and helped by the results of the HQET. We obtained the numerical results for the slope of the Isgur-Wise function, which is consistent with the experimental results, and we extracted the dependences of hadronic form factors on  $q^2$  by varying input heavy quark fragmentation function without the nearest pole dominance ansätze.

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Table 1: The values of the normalization constant  $N$  and the slope parameter  $\rho^2$  are shown with several choices for the input parameters  $\epsilon_b$  and  $\epsilon_c$ .

	$\epsilon_c = 0.04$				$\epsilon_c = 0.06$			
	$\epsilon_b = 0.004$		$\epsilon_b = 0.006$		$\epsilon_b = 0.004$		$\epsilon_b = 0.006$	
	0.06	0.08	0.1	0.1	0.06	0.08	0.1	0.1
$N$	0.938	1.068	1.190	1.306	1.105	1.223	1.338	1.449
$\rho^2$	0.705	0.646	0.609	0.582	0.896	0.844	0.810	0.785

Table 2: Comparison of the slope parameter  $\rho^2$  for the different fragmentation functions

Fragmentation Function (F. F.)	slope parameter $\rho^2$
Peterson F. F.	0.5521 – 0.8582
String F. F.	1.1240
PQCD inspired F. F.	0.6053

Figure 1: (a) The diagram of Drell-Yan Process. (b) The schematic diagram of semileptonic exclusive decay of  $B$ -meson in the parton model.

Figure 2: (a)  $q^2$  spectrum in the  $B \rightarrow De\nu$  decays. The solid line is our model prediction and the dotted line the Wirbel *et al.* model prediction [6]. We used the values of parameters, ( $\epsilon_b=0.004$ ,  $\epsilon_c=0.04$ ). (b)  $q^2$  spectrum in the  $B \rightarrow D^*e\nu$  decays. The solid line is our model prediction and the dotted line the Wirbel *et al.* model prediction [6]. We used the values of parameters, ( $\epsilon_b=0.004$ ,  $\epsilon_c=0.04$ ). The data are quoted from Ref. [10].

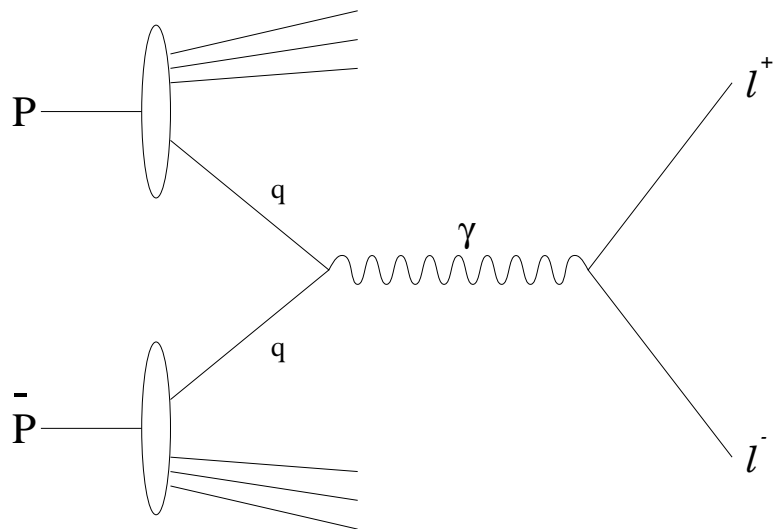


Fig. 1 (a)

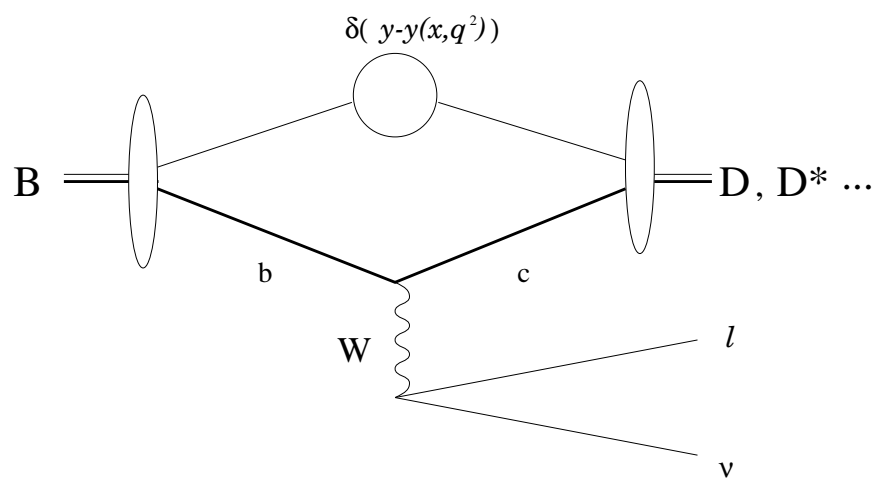


Fig. 1 (b)

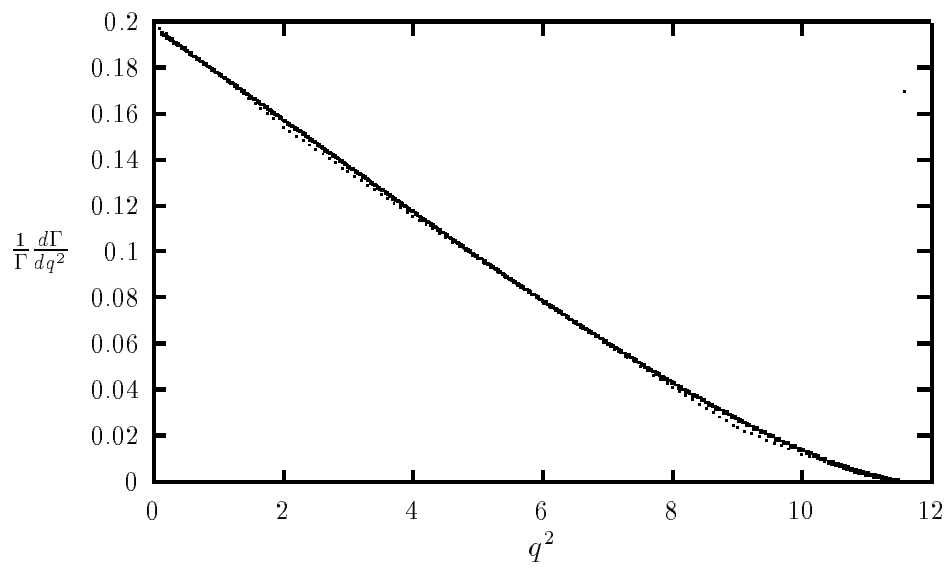
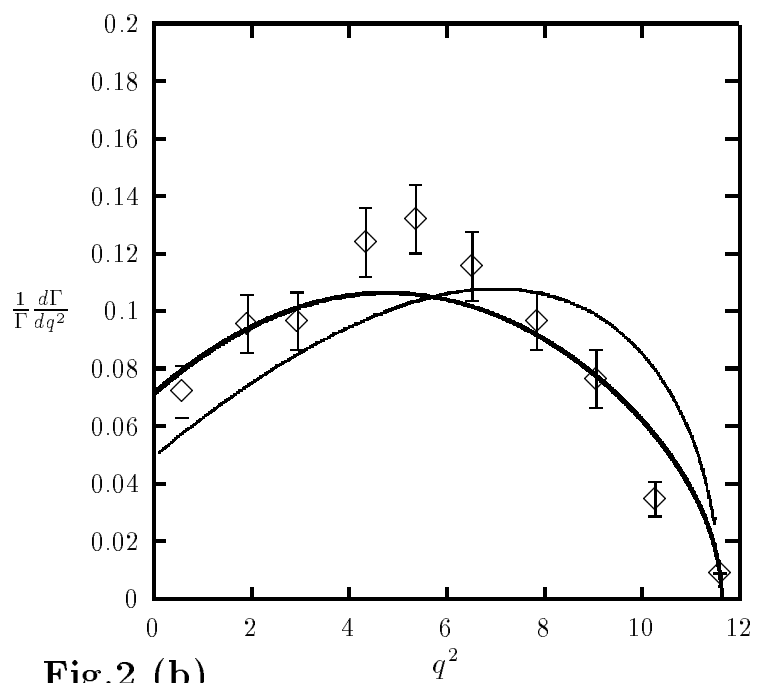


Fig.2 (a)





**Fig.2 (b)**